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DETERMINATION OF STIFFNESS OF THREE-DIMENSIONAL COMPOSITION OF ELASTIC BALLS UNDER CONDITIONS OF UNIAXIAL COMPRESSION

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Abstract. The aim of this study is to develop analytical dependencies for the uniaxial stiffness of a spatial composition of elastic balls of same diameter, considering its volumetric structure. A review of the literature was conducted regarding types of ball packings that have practical applications for describing the structure of crystals, composite materials, and ball mill loadings of various types. For calculating the stiffness of a three-dimensional composition of balls, the study is based on G. Hertz's theory of elastic ball contact. According to this theory, the relationship between compressive force and the center-to-center displacement of balls is nonlinear with an exponent of 1.5. By spatially combining individual ball contacts, the nonlinear stiffness for simple cubic and face-centered cubic packings of balls under uniaxial compression was determined. These packing types were chosen as boundary cases of regular ball packings: the former as the least dense possible packing and the latter as the densest.

Initially, the stiffness of a single layer of ball packing in a plane perpendicular to the compression force was determined by summing the parallel-connected stiffnesses of all balls. Next, the total stiffness of the spatial composition of balls compressed between two massive plates was calculated through sequential combination of the stiffnesses of all single layers along the height of the composition. Differences in the stiffness of elemental ball contacts, both between themselves and with the bounding plate layer, were taken into consideration.

As a result, formulas were derived for determining the uniaxial stiffness of the spatial ball composition for the two boundary packing types, depending on the elastic properties of the ball material and massive boundaries, the ball diameter, and the dimensions of the deformed ball composition.

The comparison of packing stiffnesses did not account for the friction coefficient due to its minor influence and its significant reduction under conditions of vibration or the presence of liquid at ball contacts. It was concluded that, firstly, the stiffness of a ball composition in a face-centered cubic packing slightly exceeds that of a simple cubic packing, within the permissible error margins of engineering calculations. Secondly, the formulas for face-centered cubic ball packing are more suitable for practical calculations. Thirdly, the results of the study can be used for modeling the stress-strain state of technological ball loadings in vibratory, planetary, and other types of mills; for modeling the behavior of layers made of solid bulk materials with approximately isometric particle shapes; and for determining the elasticity of frames in composite material fillers with significant differences in the elastic properties of their components.

Keywords: ball packing, uniaxial compression, stiffness, composite material, mill.

1. Introduction

Interest in spatial packing of balls in science arose a long time ago, since the time of K. Gauss, and has not faded to this day, looking for newer ways of application [1].

This is because a mathematical model in the form of a set of balls with the same diameter is well-suited for describing various natural structures, such as crystal structures [2]. This includes the simplest loose cubic packing, simple hexagonal, body-centered cubic, as well as the densest packings like hexagonal close-packed and face-centered cubic.

A particular area of interest is the use of ball-based models to describe the structure of composite materials [3]. These materials are becoming increasingly important in fields such as mining engineering, aerospace, aviation, shipbuilding, and other high-tech industries due to their unique properties, which result from combining diverse components, creating a synergy more effective than each component alone.

One of the most well-known composite materials is concrete and asphalt concrete products, which require high-quality aggregates, especially for applications involving high dynamic loads. This requires approximating the shape of the strong filler particles to be isometric [4], making it possible to use ball packings as a model for determining the elastic-deformation properties of the frame.

Research in mining machinery has a dedicated focus on analyzing ball load behavior in various mills [5], layer deformations of processed material in crushers, impact dampening during fragment drops on feeders, etc.

Today, various types of vibration drives of technological machines have also been developed, in particular, the vibration impact drive [6], which is actively used in the designs of ball vibration mills. The dynamic calculation of the mill as a system of bodies must necessarily take into consideration the elastic and dissipative properties of the ball load [7].

In work [8], the dependence of the force on the deformation of material rock layer in technological machines was determined as follows:

$$P_C = \frac{\lambda EF}{h^{3/2}} \cdot \left(1 + \xi \cdot signx\right) \cdot x^{3/2}, \tag{1}$$

where E is the modulus of elasticity of the material, Pa; F is the cross-sectional area of the deformed layer, m^2; h is the technological load height, m; x is the layer deformation, m.

As we can see, the dependence has a nonlinear character with a degree of 1.5, which coincides with the conclusions of G. Hertz's theory of contact deformation of elastic bodies [9].

Notably, the theory of ball packing is also used to determine the porosity of the ball load in the drum-type grinding chamber for vertical vibratory mills [10]. Here, the average porosity of the ball load is calculated as follows:

$$\bar{\varepsilon} = \varepsilon_C + \left(\varepsilon_C - \varepsilon_W\right) \cdot \left[1 - \frac{(n-2)^2}{n^2}\right], \tag{2}$$

where $\varepsilon_C = 0.259$ is the porosity at the center of the load, representing maximum-density packing; $\varepsilon_W = 0.476$ is the porosity near the walls of the grinding chamber, representing minimal-density, simple cubic packing; n is the number of balls that can fit along the chamber diameter.

This formula was derived for specific, limiting conditions.

In study [11] on the analysis of the mechanical properties of granular powders, it is noted that denser particle packing results in greater stiffness for their composite.

The authors of research [12] show that upon reaching a certain particle packing density, the stiffness of the composition begins to rise sharply due to the blocking effect of the particles' potential movements. This particle "jamming" effect is similarly corroborated in [13].

A separate study investigated the influence of friction on the structure and properties of ball packing [14]. It was observed, that system stiffness increases with higher friction coefficients, with the structure becoming more resistant to external loads, as shown under shear loads.

Thus, despite the presence of significant data in the literature regarding ball packing, elastic layer properties, and friction effects, it is necessary to summarize the existing information and derive explicit dependencies for the stiffness of spatial ball packing depending on the packing type. This would enable the prediction of the elastic properties of a ball frame for various technical applications.

The purpose of this work is to develop analytical dependencies for the uniaxial stiffness of a spatial composition of elastic balls of same diameter, considering its volumetric structure.

The idea of work is to use the force dependencies for contact interactions of elastic balls from G. Hertz's theory, followed by evaluating the distribution of contact forces in a spatially confined composition of balls.

2. Methods

An analysis of the literature sources on current research related to the topic of the article was conducted. G. Hertz's theory dependencies for the contact interaction force of elastic spheres were used. Formulas known from theoretical mechanics regarding force balance and the determination of the total stiffness of elastic elements in their series and parallel connections were applied.

3. Results and discussion

For step-by-step determination of the stiffness of an elastic ball composition, its spatial arrangement must be divided into simpler individual elements.

For each type of regular ball packing, an elementary layer of balls can be identified, with centers in a plane perpendicular to the direction of deformation.

This elementary layer can be reduced to a composition of individual balls, each occupying a certain space with surrounding voids, which are the greater the higher the porosity of packing is.

Thus, for each packing type, it is necessary to determine both the elementary volume occupied by single ball and the stiffness of contacts (from one to several), that this ball forms.

The stiffness of a single layer of packing will be determined by adding the parallel-connected stiffnesses of all balls within the layer and then sequentially combining these layers to find the total stiffness of the spatial ball composition.

We define the elementary contact stiffness of an individual ball as the resultant stiffness of all contacts it forms in the direction of deformation, considering only one side of the ball.

1. Determining stiffness for simple cubic ball packing.

In Figure 1, the simplest packing of balls is shown, where the rows of balls are aligned strictly along the axes of an orthogonal coordinate system.

This packing is the least dense among the practically possible ball packings, with a corresponding maximum porosity value of $\varepsilon_{MAX} = 0.479$.

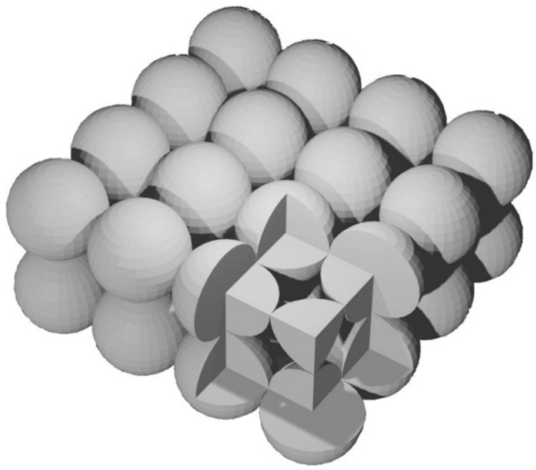

Figure 1 – Simple cubic packing of balls

We need to determine the stiffness of the ball composition under uniaxial compression conditions. For instance, let us consider the vertical direction, i.e., z-axis.

The elementary stiffness of contact between balls will be determined based on G. Hertz’s theory of elastic ball contact interaction. This theory describes a nonlinear dependence of contact force on deformation, with an exponent of 1.5 [6].

For balls of the same radius made from the same material, the nonlinear contact stiffness, according to G. Hertz's theory, is calculated as

$$C_{0,1} = \frac{E\sqrt{2R}}{3\left(1-\nu^2\right)},\ \text{Н/м}^{1.5}. \quad (3)$$

where E is Young’s modulus of the ball material, Pa; ν is Poisson’s ratio of the ball material; R is the radius of the balls, m.

It is important to note the so-called “edge effect.” When the ball composition contacts a solid bounding obstacle, which typically has a surface curvature radius much larger than that of the ball, the porosity in the layer between the obstacle and the centers of adjacent balls will approximately equal the porosity of simple cubic packing [7]. In this case, the expression for nonlinear elementary stiffness will slightly differ from equation (3):

$$C_2 = \frac{2E\sqrt{R}}{3\left(1-\nu^2\right)} = \sqrt{2}C_{0,1},\ \text{N/m}^{1.5}. \quad (4)$$

2. Determining stiffness for the densest ball packing.

A simple cubic packing of balls thicker than a single layer cannot exist in pure form due to the high degrees of freedom in ball movement, leading to spatial instability. It is observed only near solid obstacles with a radius much larger than the ball.

In dense ball packing conditions, at a certain distance from solid obstacles, the packing of balls tends to approach the densest configuration.

The densest ball packing corresponds to a minimum porosity value of $\varepsilon_{MIN} = 0.259$.

There are two known types of the densest ball packing: hexagonal and face-centered cubic. These differ only in the arrangement of the 12 contacts that each ball has but share the same minimum porosity of $\varepsilon_{MIN} = 0.259$.

It is assumed, that the stiffness of a layer of balls, under otherwise equal conditions, depends solely on the porosity value, which decreases with an increase in the average coordination number of contacts per ball and does not depend on the specific type of packing.

For convenience of analysis, we will consider the face-centered cubic packing (Fig. 2), as it has identical ball arrangements relative to any axis of the three-dimensional orthogonal coordinate system, providing uniform properties for deformation analysis.

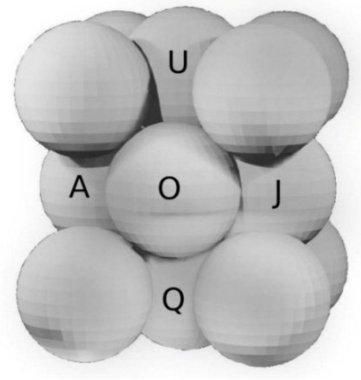

Figure 2 – Face-centered cubic packing of balls

During uniaxial deformation of such a packing, with pressure applied to a face centered by a ball *O*, the contact forces will be distributed among four balls: *A*, *J*, *U* and *Q*. The distribution scheme of contact forces is shown in Figure 3.

Here, it is necessary to calculate the proportionality coefficient between the resulting force *P*, acting on the ball in the direction of deformation *OK*, and the nonlinear deformation *OO'* in the power of 1.5 in the same direction.

Thus, the resulting force *P* is transmitted below through four contacts (only two are shown in the figure; the others lie in a plane perpendicular to the figure's plane), where each contact has a normal force component *N* and a tangential component *T*. The angle of inclination of the contact line *OA* to the compression line *OK* for this packing is 45°.

It is evident, that in an unstressed state

$$AO = D, \tag{5}$$

where *D* is the ball diameter, m.

Writing the equilibrium equations for ball O:

$$P = 4(N\cos\alpha + T\sin\alpha), \tag{6}$$

where T is the friction force, calculated using Coulomb's formula:

$$T = fN; \tag{7}$$

f is the coefficient of friction; N is the normal force, which is also determined using G. Hertz's formula (1) for the direct central contact between two balls, for simplicity:

$$N = C_{0,1} \cdot \Delta D^{1.5}, \tag{8}$$

where axial deformation of the ball contact is

$$\Delta D = AO - AO'. \tag{9}$$

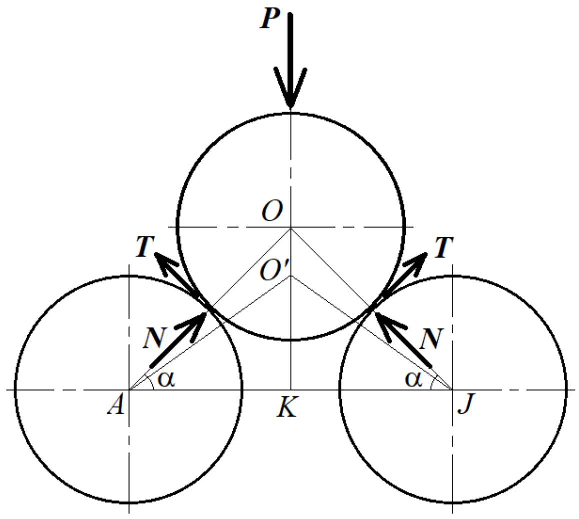

Figure 3 – Scheme for determining the reduced elementary stiffness for face-centered cubic ball packing

We denote the deformation of a single-layer ball packing as:

$$OO' = \Delta z. \tag{10}$$

From triangle AKO', the following equation is obtained:

$$(AO')^2 = AK^2 + (KO')^2; \tag{11}$$

$$(D - \Delta D)^2 = AK^2 + (KO - \Delta z)^2. \tag{12}$$

Considering that α = 45°, we have

$$AO = KO = \frac{\sqrt{2}}{2} D. \tag{13}$$

Substituting (13) into (12) and neglecting second-order small terms, we obtain:

$$\Delta z = \sqrt{2} \cdot \Delta D. \tag{14}$$

Finally, considering expressions (6), (7), (8), and (14), the nonlinear elementary stiffness of the face-centered cubic packing under uniaxial compression is determined as

$$C_{0,2} = \frac{P}{\Delta z^{1.5}} = \sqrt[4]{8} C_{0,1} (1 + f) \cong 1.68 C_{0,1} (1 + f). \tag{15}$$

The minimum value of the elementary stiffness for a face-centered cubic packing, for instance, in conditions of vibration or the presence of liquid at the ball contacts, will be

$$C_{0,2} = \sqrt[4]{8} C_{0,1} \cong 1.68 C_{0,1}. \tag{16}$$

3. Determining the total stiffness of spatial ball compositions under compression between two parallel massive plates.

The primary assumption here is the uniformity of compressive force transmitted from one horizontal layer (perpendicular to the compression line) to another. In this case, the entire ball composition behaves as a conditionally homogeneous elastic medium.

Let us introduce the following notations:

L is the length of the spatial ball composition for which stiffness is determined, m;

B is the corresponding width, m;

H is the corresponding height, m.

For both simple cubic and face-centered cubic packings, the number of balls in a single layer is determined as

$$N_0 = \frac{LB}{D^2}, \tag{17}$$

if the length and the width of spatial ball packing are multiples of the ball diameter.

Assume, that the plates are made of the same material as the balls.

Thus, there will be two ball contacts with stiffness C_2 along the height of layer:

$$N_2 = 2. \tag{18}$$

For the simple cubic packing of balls, the remaining contacts along the height will have a stiffness $C_{0,1}$. The number of such contacts, assuming the height is a multiple of the ball diameter, is:

$$N_{1,1} = \frac{H}{D} - 1. \quad (19)$$

For the face-centered cubic packing (see the schematic in Fig. 3 above), the number of contacts $C_{0,2}$ is given as

$$N_{1,2} = \sqrt{2}\frac{H}{D} - 1. \quad (20)$$

The total stiffness of spatial ball compositions will be determined first by parallel summation of the elementary stiffnesses for individual balls, followed by a sequential combination of stiffnesses along the height of the layer:
- for simple cubic packing:

$$C_{sum,1} = N_0\left(\frac{N_{1,1}}{C_{0,1}} + \frac{N_2}{C_2}\right)^{-1} = \frac{LB}{D^2}\left(\frac{N_{1,1}}{C_{0,1}} + \frac{N_2}{C_2}\right)^{-1}, \text{ N/m}^{1.5}, \quad (21)$$

or, considering equations (4), (17), (18), and (19):

$$C_{sum,1} = C_{0,1}\frac{LB}{D^2}\left(\frac{H}{D} + 0.414\right)^{-1}, \text{ N/m}^{1.5}; \quad (22)$$

- face-centered cubic packing:

$$C_{sum,2} = \frac{LB}{D^2}\left(\frac{N_{1,2}}{C_{0,2}} + \frac{N_2}{C_2}\right)^{-1}, \text{ N/m}^{1.5}. \quad (23)$$

or, considering equations (4), (16), (17), (18), and (19):

$$C_{sum,2} = C_{0,1}\frac{LB}{D^2}\left(0.842\frac{H}{D} + 0.819\right)^{-1}, \text{ N/m}^{1.5}. \quad (24)$$

For height-to-diameter ratios (*H/D*) = 5, 10 and 20, the stiffness $C_{sum,2}$ exceeds $C_{sum,1}$ by 8%, 13%, and 16%, respectively.

In this comparison, the friction factor was not considered, primarily because of its minor influence and, secondarily, due to the questionable validity of accounting for friction in face-centered cubic packing while ignoring it in simple cubic packing.

From a practical perspective, simple cubic packing is rarely encountered in real ball compositions due to its spatial instability. However, face-centered cubic packing is the most realistic configuration, especially under vibration conditions.

4. Applications of the research results.

The findings of this research can be applied primarily in projects related to using material crushing, mixing, and activation technologies.

For instance, in the design of vibratory mills, determining the stiffness of ball loading will enable the calculation of energy transfer from the chamber to the grinding balls.

In the production of powder materials from different components, selecting appropriate elastic properties for the ball loading can enhance the uniform distribution of components and prevent the formation of agglomerates.

Accurately determining the stiffness of ball loading will also help predict the wear of balls and mill linings, optimize maintenance intervals, reduce operational costs, and extend the service life of the equipment.

Within the framework of "smart" manufacturing, accounting for the stiffness of the loading and monitoring changes in this parameter will allow seamless integration into automated mill control systems to improve their efficiency.

4.Conclusions

1. Analytical dependencies were derived for determining the stiffness of spatial compositions of elastic balls for simple cubic and face-centered cubic packings, which represent opposite boundary models in terms of layer porosity.

2. It was established that the stiffness of the face-centered cubic ball composition slightly exceeds that of the simple cubic packing, remaining within the acceptable margin of error for engineering calculations.

3. The formulas for face-centered cubic ball packing are more suitable for practical calculations.

4. The results of this study can be applied in modeling the stress-strain state of the technological ball load in vibratory, planetary, and other types of mills, for modeling layers of solid granular materials with particle shapes approximating isometric, and for determining the elasticity of filler frameworks in composite materials with significantly different component elasticity characteristics.

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ВИЗНАЧЕННЯ ЖОРСТКОСТІ ОБ'ЄМНОЇ КОМПОЗИЦІЇ ПРУЖНИХ КУЛЬ В УМОВАХ ОДНОВІСНОГО СТИСКУ

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Анотація. Метою даної роботи є аналітичні залежності одновісної жорсткості просторової композиції пружних куль однакового розміру від параметрів такої композиції, беручі до уваги її об'ємну структуру. Проаналізовано літературні джерела щодо типів укладок куль, які мають практичне застосування для опису будови кристалів, композитних матеріалів, кульового завантаження млинів різних типів. Для розрахунку жорсткості тривимірної композиції куль взято за основу теорію контакту пружних куль Г. Герца, за якою характер залежності зусилля стискання від зближення центра куль має нелінійний характер зі ступенем 1,5. Поєднуючи у просторі окремі контакти куль, визначено нелінійну жорсткість для простої кубічної та гранецентрованої кубічної укладок куль у випадку одновісного стиску. Ці типи укладок взято як граничні випадки регулярних укладок куль, зокрема, першу – як найменш щільну з можливих укладок, а другу – як найбільш щільну. Спочатку визначено жорсткість одиночного шару укладки куль в площині, що перпендикулярна до напрямку сили стиску, шляхом додавання паралельно поєднаних жорсткостей усіх куль. На наступному кроці, визначено сумарну жорсткість просторової композиції куль при стисканні між двома масивними плитами через послідовне поєднання жорсткостей усіх одиночних шарів за висотою композиції. Взято до уваги відмінності жорсткостей елементних контактів куль між собою та з обмежуючою шар плитою. В результаті, отримано формули для визначення одновісної жорсткості просторової композиції куль для двох граничних типів укладок в залежності від пружних властивостей матеріалу куль та масивних перешкод, діаметру куль та габаритів деформованої композиції куль. Під час порівняння жорсткостей укладок не враховувався коефіцієнт тертя, як через його невеличкий вплив, так і через його різке зменшення в умовах вібрації або наявності рідини на контактах куль. Зроблено висновки про те, що, по-перше, жорсткість композиції куль для гранецентрованої кубічної укладки несуттєво перевищує жорсткість для простої кубічної укладки, практично в межах припустимої похибки інженерних розрахунків. По-друге, формули для гранецентрованої кубічної укладки куль більш прийнятні для практичних розрахунків. По-третє, результати роботи можуть бути використані для моделювання напружено-деформованого стану технологічного кульового завантаження вібраційних, планетарних та інших типів млинів, для моделювання поведінки прошарків із твердих сипких матеріалів з формою шматків, наближеної до ізометричної, а також для визначення пружності каркасу заповнювачів композитних матеріалів із суттєвою відмінністю пружних характеристик компонентів.

Ключові слова: укладка куль, одновісний стиск, жорсткість, композитний матеріал, млин.